

Shell-model studies on exotic nuclei around ^{132}Sn

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Abstract

The study of exotic nuclei around ^{132}Sn is a subject of current experimental and theoretical interest. Experimental information for nuclei in the vicinity of ^{132}Sn , which have been long inaccessible to spectroscopic studies, is now available thanks to new advanced facilities and techniques. The experimental data which have been now become available for these neutron-rich nuclei may suggest a modification in the shell structure. They are, in fact, somewhat different from what one might expect by extrapolating the existing results for $N < 82$, and as a possible explanation a change in the single-proton level scheme has been suggested. The latter would be caused by a more diffuse nuclear surface, and could be seen as a precursor of major effects which should show up at larger neutron excess. New data offer therefore the opportunity to test the shell model and look for a possible evolution of shell structure when going toward neutron drip line. This is stimulating shell-model studies in this region. Here, we present an overview of recent shell-model studies of ^{132}Sn neighbors, focusing attention on those calculations employing realistic effective interactions.

Key words:

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1. Introduction

The region of nuclei around doubly-magic ^{132}Sn is currently a subject of a certain theoretical and experimental interest. New advanced facilities and techniques, such as the advent of radioactive ion beams, allow to access to new data [1,2,3], that give the opportunity to test theoretical models.

In particular, it is important to study in this region, by way of microscopic approaches, the possible evolution of shell structure when going toward proton or neutron drip

lines [4,5]. In fact, recent works evidence that in $N = 82$ isotones the so-called “shell-quenching” phenomenon seems to play a fundamental role to reproduce the solar system r -process abundances ($N_{r,\odot}$) [6]. This is stimulating shell-model studies in this region, focusing particular attention on the two-body matrix elements (TBME) of the residual interaction.

In recent years, the derivation of the shell-model TBME from realistic nucleon-nucleon (NN) potentials has proved to be a reliable approach to microscopic shell-model calculations [7,8]. The success achieved by these calculations in different mass regions gives a clearcut answer to the long-standing problem of how accurate description of nuclear structure properties can be provided by realistic shell-model interactions, and opens the way to a more fundamental approach to the nuclear shell model than the traditional one, which makes use of empirical TBME with several parameters.

The paper is organized as follows. In Sec. II we give a short description of how the short-range repulsion of a realistic NN potential is renormalized before to be employed in the derivation of the effective shell-model interaction. In particular, we will focus attention on a recent approach, the so-called $V_{\text{low-k}}$ one [9], that allows to derive a low-momentum NN realistic potential which preserves exactly the onshell physics of the original potential. In Sec. III a summary of the derivation of the shell-model effective hamiltonian H_{eff} is presented with some details of our calculations. In Sec. IV we present and discuss results obtained employing realistic shell-model interactions for nuclei with valence nucleons outside the doubly-magic ^{132}Sn core. Some concluding remarks are given in Sec. V.

2. The renormalization of the short-range repulsion

Because of the strong repulsive core in the short-range region, which is a feature common to all modern NN potentials, the latter cannot be used directly in the derivation of shell-model effective interaction within a perturbative approach, that is the standard procedure. So, as mentioned before, realistic potentials have to be renormalized first. The standard way to renormalize the short-range repulsion is to resort to the theory of the Brueckner reaction matrix G , that provides to sum up the infinite series of ladder diagrams whose interaction vertices are the NN interaction itself. The G matrix is defined [10] by the integral equation:

$$G(\omega) = V_{NN} + V_{NN} Q_{2p} \frac{1}{\omega - Q_{2p} T Q_{2p}} Q_{2p} G(\omega) \quad , \quad (1)$$

where V_{NN} represents the NN potential, T denotes the two-nucleon kinetic energy, and ω is an energy variable (the so-called starting energy), given by the energy of the incoming nucleons. The two-body Pauli exclusion operator Q_{2p} prevents double counting, namely the intermediate states allowed for G must be outside of the chosen model space. Thus the Pauli operator Q_{2p} is dependent on the model space, and so is the G matrix.

Inspired by the effective theory, an alternative approach based on the renormalization group (RG) has been recently introduced to renormalize the short-range repulsion introducing a cutoff momentum Λ , that decouples the fast and slow modes of the original V_{NN} .

Let us now outline briefly the derivation of this low-momentum potential $V_{\text{low-k}}$ [9]. The repulsive core contained in V_{NN} is smoothed by integrating out the high-momentum modes of V_{NN} down to Λ . This integration is carried out with the requirement that the deuteron binding energy and low-energy phase shifts of V_{NN} are preserved by $V_{\text{low-k}}$. This is achieved by the following T -matrix equivalence approach. We start from the half-on-shell T matrix for V_{NN}

$$T(k', k, k^2) = V_{NN}(k', k) + \mathcal{P} \int_0^\infty q^2 dq V_{NN}(k', q) \frac{1}{k^2 - q^2} T(q, k, k^2) , \quad (2)$$

where \mathcal{P} denotes the principal value and k , k' , and q stand for the relative momenta. The effective low-momentum T matrix is then defined by

$$T_{\text{low-k}}(p', p, p^2) = V_{\text{low-k}}(p', p) + \mathcal{P} \int_0^\Lambda q^2 dq V_{\text{low-k}}(p', q) \frac{1}{p^2 - q^2} T_{\text{low-k}}(q, p, p^2) , \quad (3)$$

where the intermediate state momentum q is integrated from 0 to the momentum space cutoff Λ and $(p', p) \leq \Lambda$. The above T matrices are required to satisfy the condition

$$T(p', p, p^2) = T_{\text{low-k}}(p', p, p^2); \quad (p', p) \leq \Lambda . \quad (4)$$

The above equations define the effective low-momentum interaction $V_{\text{low-k}}$, and it has been shown [9] that they are satisfied by the solution:

$$V_{\text{low-k}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} + \dots , \quad (5)$$

which is the well known Kuo-Lee-Ratcliff (KLR) folded-diagram expansion [11,12], originally designed for constructing shell-model effective interactions. In Eq. (5) \hat{Q} is an irreducible vertex function whose intermediate states are all beyond Λ and \hat{Q}' is obtained by removing from \hat{Q} its terms first order in the interaction V_{NN} . In addition to the preservation of the half-on-shell T matrix, which implies preservation of the phase shifts, this $V_{\text{low-k}}$ preserves the deuteron binding energy, since eigenvalues are preserved by the KLR effective interaction. For any value of Λ , the low-momentum potential of Eq. (5) can be calculated very accurately using iteration methods. Our calculation of $V_{\text{low-k}}$ is performed by employing the iteration method proposed in [13], which is based on the Lee-Suzuki similarity transformation [14].

The main result is that $V_{\text{low-k}}$ is a smooth potential which preserves exactly the onshell properties of the original V_{NN} , and is suitable to be used directly in nuclear structure calculations. In the past few years, $V_{\text{low-k}}$ has been fruitfully employed in microscopic calculations within different perturbative frameworks such as the realistic shell model [15,16,17,18], the Goldstone expansion for doubly closed-shell nuclei [19,20,21], and the Hartree-Fock theory for nuclear matter calculations [22,23].

3. The derivation of the shell-model effective potential

In the framework of the shell model, an auxiliary one-body potential U is introduced in order to break up the nuclear hamiltonian as the sum of a one-body component H_0 ,

which describes the independent motion of the nucleons, and a residual interaction H_1 :

$$H = \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{i < j} V_{ij} = T + V = (T + U) + (V - U) = H_0 + H_1 \quad . \quad (6)$$

Once H_0 has been introduced, a reduced model space is defined in terms of a finite subset of H_0 's eigenvectors. The diagonalization of the many-body hamiltonian (6) in an infinite Hilbert space, that is obviously unfeasible, is then reduced to the solution of an eigenvalue problem for an effective hamiltonian H_{eff} in a finite space.

The standard approach is to derive H_{eff} by way of the time-dependent perturbation theory [11,12]. Namely, H_{eff} is expressed through the KLR folded-diagram expansion in terms of the vertex function \hat{Q} -box, which is composed of irreducible valence-linked diagrams. The \hat{Q} -box is composed of one- and two-body Goldstone diagrams through a certain order in V [24], where V is the renormalized input potential. Once the \hat{Q} -box has been calculated, the series of the folded diagrams is summed up to all orders using the Lee-Suzuki iteration method [14].

The hamiltonian H_{eff} contains one-body contributions, which represent the effective single-particle (SP) energies. In realistic shell-model calculations it is customary to use a subtraction procedure [25] so that only the two-body terms of H_{eff} are retained - the effective interaction V_{eff} - and the SP energies are taken from the experimental data. This is what has also been done in the calculations reported in the following section. The single-proton and single-neutron energies have been taken from the experimental spectra of ^{133}Sb and ^{133}Sn [26], that can be described just as one proton and one neutron diving in the mean field generated by the ^{132}Sn doubly-closed core. For sake of completeness, it is important to mention that experimentally the proton $2s_{1/2}$ level is missing and the neutron $0i_{13/2}$ level is unbound. The analysis of the first excited $J^\pi = 10^+$ in ^{134}Sb allows to estimate the SP energy of neutron $0i_{13/2}$ level to be 2.694 ± 0.2 MeV [27], while a study of odd $N = 82$ isotones suggests that the SP proton $2s_{1/2}$ level should lie around 2.8 MeV excitation energy [28].

4. Realistic shell-model calculations

Here, we present some selected results employing realistic shell-model V_{eff} s for nuclei with valence nucleons outside ^{132}Sn core. The V_{eff} s are derived from the CD-Bonn realistic NN potential [29] using as renormalization approach both the G -matrix and the $V_{\text{low-k}}$ ones. Calculations employing a G -matrix derived from the CD-Bonn potential have been widely performed by the Oslo group and their coworkers [2,3,7,30,31] with a remarkable success. The V_{eff} is derived within the folded-diagram approach described in section 3, including in the \hat{Q} -box diagrams up to the third order in G and intermediate states with at most $2\hbar\omega$ excitation energy [32].

The region of nuclei around ^{132}Sn core has been studied in recent years also by the Naples-Stony Brook group within the framework of the $V_{\text{low-k}}$ renormalization procedure. In such a case, the shell-model effective interaction is obtained starting from a $V_{\text{low-k}}$ derived from the CD-Bonn potential, with a cutoff momentum $\Lambda = 2.2 \text{ fm}^{-1}$, that is a value able to preserve all the two-nucleon physics of the original potential up to the anelastic threshold and small enough so to give a reasonably smooth potential. Then the \hat{Q} -box has been calculated including diagrams up to second order in $V_{\text{low-k}}$ using

intermediate states composed of all possible hole states and particle states restricted to the five shells above the ^{132}Sn Fermi surface. This guarantees the stability of the V_{eff} TBME when increasing the number of intermediate particle states.

In both approaches an harmonic-oscillator basis with an oscillator parameter $\hbar\omega = 7.88$ MeV has been employed.

Let us now come to the results of the calculations and their comparison with experimental data.

First, it is worth to point out that a fundamental test for the reliability of the matrix elements of V_{eff} are the systems with two valence nucleons outside di closed-shell core. In the present case, the test is the reproduction of the spectra of ^{134}Te - two protons outside ^{132}Sn - ^{134}Sn , and ^{134}Sb , that are two neutrons and one proton and one neutron outside ^{132}Sn , respectively.

In Tables 1,2,3 the experimental and theoretical low-lying spectra of ^{134}Te , ^{134}Sn , ^{134}Sb are reported.

Table 1

Experimental energy levels up to 3 MeV [26] for ^{134}Te compared to the calculation with CD-Bonn potential through G -matrix [31] and $V_{\text{low-k}}$ [8] renormalization procedures, respectively.

J^π	Experiment	G -matrix	$V_{\text{low-k}}$
0_1^+	0.0	0.0	0.0
2_1^+	1.28	1.21	1.33
4_1^+	1.57	1.48	1.61
6_1^+	1.69	1.61	1.75
6_2^+	2.40	2.17	2.45
2_2^+	2.46	2.45	2.67
4_2^+	2.55	2.45	2.63
1_1^+	2.63	2.41	2.67
3_1^+	2.68	2.54	2.68
5_1^+	2.73	2.54	2.68
2_3^+	2.93	3.06	3.27

Table 2

Experimental observed energy levels [26] for ^{134}Sn compared to the calculation with CD-Bonn potential through G -matrix [7] and $V_{\text{low-k}}$ [8] renormalization procedures, respectively.

J^π	Experiment	G -matrix	$V_{\text{low-k}}$
0_1^+	0.0	0.0	0.0
2_1^+	0.726	0.775	0.733
4_1^+	1.073	1.116	1.016
6_1^+	1.247	1.258	1.125
8_1^+	2.509	2.463	2.545

From the inspection of Tables 1,2, it is evident that the agreement between theory and experiment is very good for the identical particle channel, both with G -matrix and

$V_{\text{low-k}}$ approach. It is worth to note that theoretical results do not differ so much when using the two different renormalization procedures.

Table 3

Experimental energy levels up to 1 MeV [2] for ^{134}Sb compared to the calculation with CD-Bonn potential through G -matrix [2] and $V_{\text{low-k}}$ [18] renormalization procedures, respectively.

J^π	Experiment	G -matrix	$V_{\text{low-k}}$
0_1^-	0.0	0.0	0.0
1_1^-	0.013	0.329	0.052
7_1^-	0.279	0.392	0.407
2_1^-	0.330	0.406	0.385
3_2^-	0.383	0.581	0.419
5_1^-	0.442	0.604	0.494
4_1^-	0.555	0.710	0.621
6_1^-	0.617	0.849	0.727
1_2^-	0.885	1.268	0.868
2_2^-	0.935	1.051	0.958

The study of ^{134}Sb low-lying spectrum evidences a differ situation respect the identical-particle case. A less better agreement with experimental data is obtained employing the V_{eff} TBME derived with the G -matrix approach. This deficiency is also propagated in the theoretical spectrum of ^{135}Sb obtained with G -matrix renormalization procedure, as it can be seen in Table 4. In Ref.[3], in order to obtain a better agreement with experimental spectrum of ^{135}Sb , it was pointed out that a downshift of the proton $d_{5/2}$ level with respect to the $g_{7/2}$ one by 300 keV, as a possible collective influence of a neutron skin [30], turned out to be necessary, but did not help for ^{134}Sb [30,2].

We have verified that the differences between the results obtained with G -matrix and those with $V_{\text{low-k}}$, that are in quite good agreement with experiment, should be traced mainly to the different dimension of the intermediate state space. In fact, it has been found that including intermediate states only up to $2\hbar\omega$ excitation energy in the calculation of the second-order \hat{Q} -box, with the $V_{\text{low-k}}$ as input potential, the results become similar, as in the case of the identical-particle channel.

Table 4

Experimental energy levels up to 1 MeV [3] for ^{135}Sb compared to the calculation with CD-Bonn potential through G -matrix [3] and $V_{\text{low-k}}$ [33] renormalization procedures, respectively.

J^π	Experiment	G -matrix	$V_{\text{low-k}}$
$7/2_1^+$	0.0	0.0	0.0
$5/2_1^+$	0.282	0.527	0.391
$3/2_1^+$	0.440	0.438	0.509
$11/2_1^+$	0.707	0.662	0.750
$9/2_1^+$	0.798	0.947	0.813
$7/2_2^+$	1.014	1.135	0.938
$9/2_2^+$	1.027	1.165	1.108

This confirms the fact that results do not depend so much on the renormalization technique. However, it is worth to note that, because $V_{\text{low-k}}$ does not depend on the Pauli-blocking operator Q as the G -matrix, using $V_{\text{low-k}}$ one can easily employ a larger number of intermediate states and obtaining consequently better results.

5. Concluding remarks

Here, we have presented selected results of some shell-model studies of nuclei with valence-nucleons outside doubly-closed shell core ^{132}Sn , where realistic effective shell-model interactions have been employed. In particular, we have focused the attention on few valence-nucleons nuclei which are most appropriate for a stringent test of the two-body matrix elements. The latter have been derived by means of a \bar{Q} -box folded-diagram method from the CD-Bonn potential, renormalized both by use of the G -matrix and $V_{\text{low-k}}$ approaches. Results are in a good agreement with experiment and, in particular, do not depend strongly on the renormalization technique employed, except a slightly difference in the neutron-proton interaction. To conclude, ...

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